Study of the Dynamics of Spin-Polarized Vertical Cavity Surface Emitting Lasers Using Largest Lyapunov Exponent

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ABSTRACT
Polarization dynamics of spin polarized Vertical Cavity Surface Emitting Lasers (VCSELs) has been the subject of intense research work. Apart from the interesting physics involved, polarization dynamics of VCSELs offer a platform for a wide range of applications. Recently, the potential of the use of Quantum Dot (QD) VCSELs has been highlighted. Regardless of the active material, reliable resolution of the dynamics is important. Two methods are primarily used, namely Bifurcation Theory (BT) and the Largest Lyapunov Exponent (LLE). Among the two, BT is more popular within the optoelectronics community by virtue of its simplicity. The method can determine in a fast and trackable manner the *kind* of the nonlinear dynamics. However, it cannot *quantify* the strength of nonlinear dynamics. This is possible with the LLE method which can be beneficial in some cases where an asymmetry in the dynamics of the system might exist. Our recent work on the dynamics of spin polarized QD-VCSELs that accounts for both the effects of Ground States (GS) and Excited States (ES) has concluded that the time evolution of the ES and GS is asymmetric. This is studied here by means of LLE.
Keywords: VCSEL, polarization dynamics, quantum dots, largest Lyapunov exponent.

1. INTRODUCTION

Polarization dynamics in spin VCSELs offer a platform for a plethora of applications that include polarization switching, circular dichroism spectroscopy and biological studies among others [1], [2]. The output polarization of spin VCSELs is controlled with the injection of spin-polarized electrons. The coupling of the spin injected carriers with the circularly polarized optical field formulate the basis for nonlinear phenomena. Then, nonlinearities can evolve in these devices without the requirement for optical injection or optical feedback. The use of quantum dots (QDs) as the active material in spin VCSEL offers more degrees of freedom for the manipulation of the operational dynamics [3]. For the resolution of the dynamics, BT monopolizes the interest of the optoelectronics community, and justifiably so, as it can provide a full mapping of the dynamics, from stability and oscillatory behavior to chaos. BT identifies the kind of dynamics while the particulars of the nonlinear behavior can be deduced from study of the time series.

An alternative method that can be used to identify chaotic behaviour is LLE. The LLE method is particularly suited for identifying as well as quantifying the chaotic behaviour.

The purpose of the present contribution is twofold, on the one hand we wish to raise the issue of the integration dynamics and on the other to validate and highlight the potential of LLE through a comparative study with BT.

2. LLE AND BIFURCATION THEORY

A bifurcation diagram of a dynamical system shows the possible long-term values (equilibria/fixed points, periodic, or chaotic orbits) of the system as a function of a bifurcation parameter. Here for the generation of the bifurcation diagrams we adopt a simple approach where we identify the local extrema (maxima and minima) of the time series of the fields of optically injected spin QD VCSELs and subsequently plot these as a function of the injection control parameter.

A very useful tool for the characterization of the dynamics of an optically injected spin VCSEL is LLE. LLE is a scalar quantity that measures the divergence of the trajectories of initially nearby points in the phase space as the system evolves in time. For the calculation of the LLE consider two nearby trajectories *a* and *b*, with small initial distance d_0 (of the order of $\sim 10^{-8}$) that evolve in time. At each time step, the new distance d_i is measured and the quantity $ln(d_i/d_0)$ divided by the time step of integration, is calculated. Then, after the time evolution of the system the LLE is defined as[4]:

$$LLE_{\xi} = \frac{\sum_{i=1}^{N} \ln\left(\frac{d_i^{\xi}}{d_{i-1}^{\xi}}\right)}{N(time\ step)}$$

where N is the number of steps along the time series. Ideally, a negative LLE indicates a fixed point, and a positive LLE indicates chaotic states while zero LLE indicate periodic and quasi-periodic states.

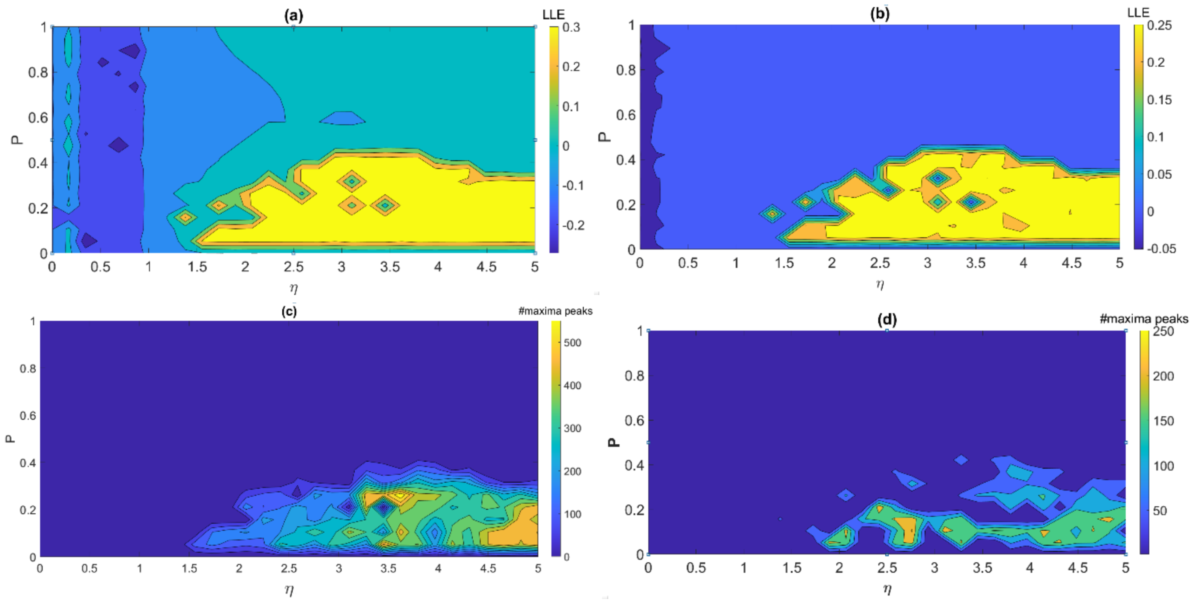

Figure 1. Calculated maps P- η for two state QD-spin-VCSEL for t = 80 ns:
a) LLE ES b) LLE GS c) BT ES, d) BT GS.

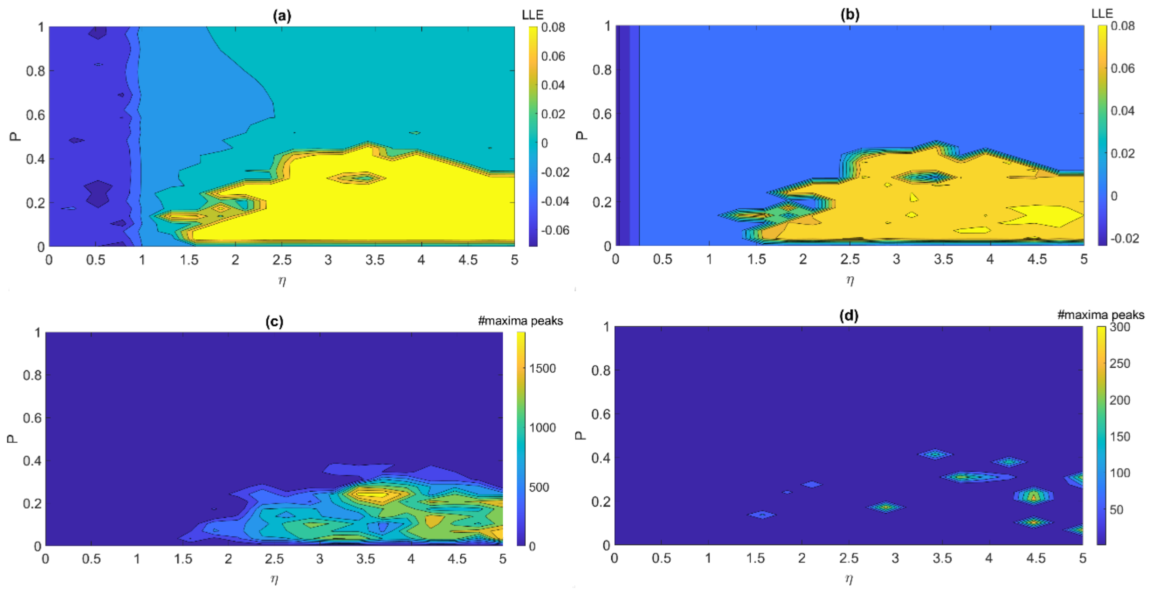

Figure 2. Calculated maps P- η for two state QD-spin-VCSEL for t = 200 ns:
a) LLE ES b) LLE GS c) BT ES, d) BT GS.

3. QD SPIN VCSEL

Here we use an elaborated model of the QD spin VCSEL model [5], to account for the effects of the Excited States on the dynamics of the system. The details of the model are presented in [6]. In QD spin VCSELs with emission from both ES and Ground State (GS) emission, the spin-relaxation process between the spin polarized carriers, spin-up (-) and spin-down (+), in the wetting layer is characterized by rate γ_{WL}. The capture rate to the ES level is γ_0. Once the spin polarized carriers have been captured into the spin-up(down) ES they can relax at the spin-up(down) GS level with an intra-dot relaxation rate γ_{21}. Intra-dot spin-relaxation process can occur from spin-up(down) ES to spin-down(up) ES as well as from spin-up(down) GS to spin-down(up) GS at a rate γ_{QD}. Lasing occurs via transitions from the ES or GS, to the valence band (VB) emitting right (E_{ES}^+, E_{GS}^+) and left (E_{ES}^-, E_{GS}^-) circularly polarized electric fields at two distinct wavelengths. The right and left circularly polarized fields are coupled via birefringence rate γ_p and gain anisotropy γ_a [7]. The set of parameters that characterize the spin VCSEL is completed with the cavity loss rate κ, and the linewidth enhancement factor α. Here our interest is focused on solitary spin QD VCSELs that are optically injected with spin polarized carriers. Then the control quantities of interest are the pump ellipticity defined as $P = (\eta^+ - \eta^-) / (\eta^+ + \eta^-)$ where is the pump of spin up (down) carriers, and $\eta = \eta^{+} + \eta^{-}$ is the total pump intensity. The dynamics of QD spin VCSELs are commonly presented with stability maps in the *(P, η)* space.

3.1 Time evolution effects on the stability maps of QD spin VCSELs

The modified model of [5] is used here to study the effect of the time evolution of the system on the particulars of the dynamics. Figure 1 shows the (*P, η*) maps for *t* = 80 ns integration time of the system calculated for both LLE and BT. The set of parameters used for Fig. 1 and subsequent figures are: $\gamma_p = 20$ ns^{-1}, $\gamma_{21} = 150$ ns^{-1}, $\gamma_s = 10$ ns^{-1}, $\kappa = 250$ ns^{-1}, $\alpha = 3$ for GS and $\alpha = 1.5$ for ES, $\gamma_n = 1$ ns^{-1} and $\gamma_o = 400$ ns^{-1}. Figure 2 shows the (*P, η*) maps calculated for both LLE and GS, this time the integration time is *t* = 200 ns. It is evident that the LLE method is more efficient in visualizing the areas of chaotic behaviour than BT. Although the topography of both LLE and BT maps is similar, there exist some differences especially for GS transitions for longer integration time.

Evidently the time the system is allowed to evolve impacts the value of the LLE and the amount of the dynamics. Although the dynamics of ES and GS exhibit similar values of LLE, these are not identical. In other words there exists an asymmetry that persists even if the system evolves for longer times as shown in Fig. 3.

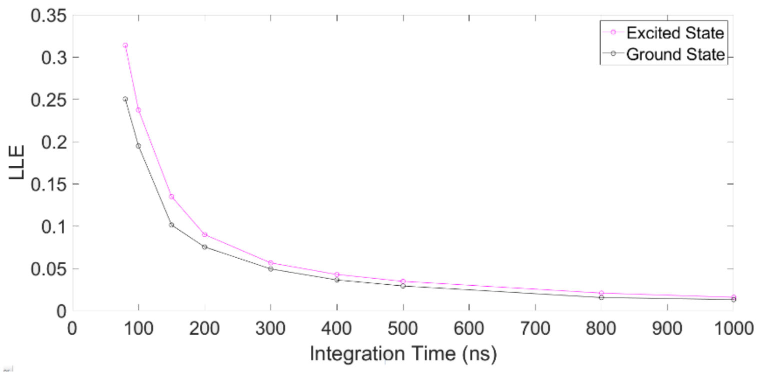

Figure 3. LLE change with integration time.

4. CONCLUSIONS

We studied the dynamics of dual emission spin QD VCSELs using the LLE and the BT. The LLE method has the advantage over BT that it can quantify the chaotic dynamics while BT can only identify the kind of dynamics. The time the system is allowed to evolve in time affects significantly the LLE value. The dynamics of the ES and GS, are always of the same kind but differ in magnitude.

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